Physical simulation based technique for determining continuous maps between meshes

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The classification theorem of closed surfaces states, that any connected compact orientable surface is homeomorphic to the sphere or connected sum of n tori, for $n \geq 1$. If $X, Y \subset \mathbb{R}^3$ are topologically equivalent surfaces, then there exists an $H : X \to Y$ bijection between the two surface such H and H^{-1} are continous functions. Let us suppose, that f and g are the parameterizations of X and Y are given, i.e. for some $I \subset \mathbb{R}^2$ two-dimensional interval $f : I \to X$ and $g : I \to Y$ bijections are known. The so-called homotopy between the functions f and g is defined as an $h : [0,1] \times I \to \mathbb{R}^3$ continuous map, such $h(0,x) = f(x)$ and $h(1,x) = g(x)$. As f and g are parameterization of the topologically equivalent surfaces, then h can be interpreted as a continuous deformation between the surface X and Y. In our research we are looking for this type of transition function for given X and Y surfaces. In computer graphics a surface is mostly represented by a triangular mesh, i.e. a set of vertices and the vertex indices of triangular surface elements. Since the parameterization of the surface is unknown, it is not an easy to define a transition between X and Y. The fact, that a given surface is topologically equivalent to the sphere (or n-tori) can be checked quite easily, but commonly defining the homeomorphism between surfaces or the homotopy between parameterizations is a difficult task. On the other hand, if we knew the parameterization of the surfaces, defining a homeomoprhism would be much easier. Therefore we invented a method, that can determine the parameterization of meshes that are homeomorphic to the sphere. The main idea came from the real world, we simulate the motion of the surface particles of an air balloon, because if the internal pressure is large enough, the balloon surface is similar to a sphere. Let us consider the vertices, edges and the mesh to be particles, springs and surface of a soft body, respectively. Then we can simulate the pressure acting in the direction of face normals. As the faces move outwards, the volume of the body increases, therefore the pressure decreases. The springs do not let the adjacent vertices to move far away from each other. Obviously, if each vertices displaced to a sphere, then the spring force and pressure force act in the opposite direction, therefore this is a state of equilibrium. If the simulation is successful, then we get a homeomorphism between a given surface and the sphere, therefore we can easily determine the parameterization and the transition function between surfaces. We will present our results in physical simulation, mesh parameterization and determining homeomorphisms and transitions between meshes.

References

[1] K. Crane, U. Pinkall, P. Schröder. Robust Fairing via Conformal Curvature Flow, *ACM Transactions on Graphics (TOG) Volume 32 Issue 4*, 2013.

[2] C. Ericson. Real-Time Collision Detection. *Elsevier*, 2005.

[3] G. Fábián, L. Gergó. Planning data structure for space partitioning algorithms, *International Journal of Applied Mathematics and Informatics*, revised, 2016.

[4] S. Ghali. Introduction to Geometric Computing, *Springer-Verlag London Limited*, 2008.

[5] J. M. Lee. Introduction to Topological Manifolds, *Springer*, 2000.